Madras College Maths Department
Higher Maths
E&F 1.3 Functions and Graphs

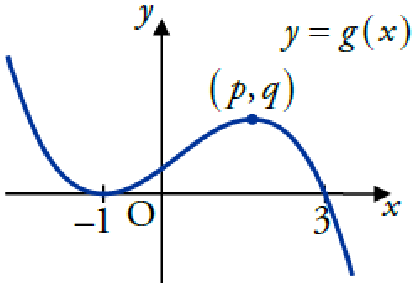
Page	Topic	Textbook
2-8	Transformation of Graphs	Page 58 Ex 3A All questions
9-10	Competing The Square	Page 60 Ex 3B Q1-4
11-13	Sketching Parabolas	Page 65 Ex 3C Q1aceg 2aceg 3, 5ace, 6, 8, 9, 11, 12
14-15	Graphs of exponential functions	Page 68 Ex 3D Q1ace 2acef 3aceg 4-6
16	Logarithmic Graphs	Page 73 Ex 3E Q1ac 2ac 3aceg 4, 5
17-18	Sketching Trigonometric Graphs	Page 77 Ex 3F Q1ace 2, 4, 5, 7
19-23	Sets and Functions – The Domain and Range of a Function	Page 85 Ex 4A All questions
24-25	Composite Functions	Page 87 Ex 4B All questions
26-28	Inverse Functions	Page 89 Ex 4C Q1, 2
29	Extension to Logarithmic and Exponential Functions	Page 91 Ex 4D All questions
30-34	Practice Unit Assessments	
35-37	Homework	

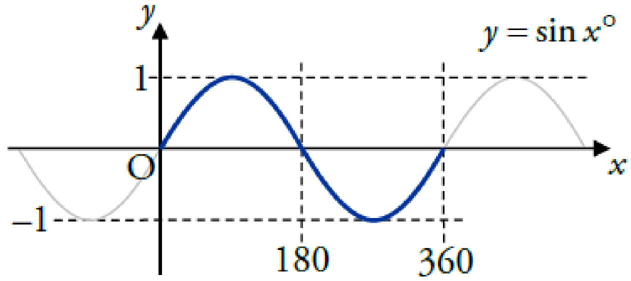
Written solutions for each exercise are available at
http://madrasmaths.com/courses/higher/revison_materials_higher.html

You should check your solutions at the end of each exercise and ask your teacher or attend study support if there any problems.

Graph Transformations

The graphs below represent two functions. One is a cubic and the other is a sine wave, focusing on the region between 0 and 360.





In the following pages we will see the effects of three different "transformations" on these graphs: translation, reflection and scaling.

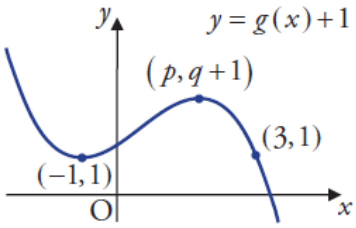
Translation

A **translation** moves every point on a graph a fixed distance in the same direction. The shape of the graph does not change.

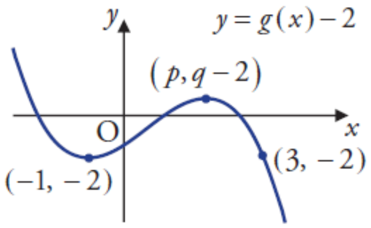
Translation parallel to the y-axis

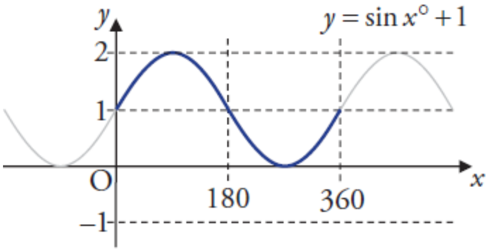
$f(x)+a$ moves the graph of $f(x)$ up or down. The graph is moved up if a is positive, and down if a is negative.

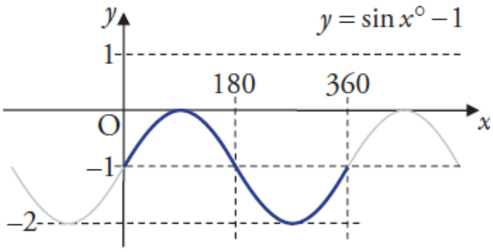
a is positive



a is negative



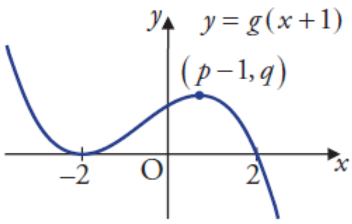




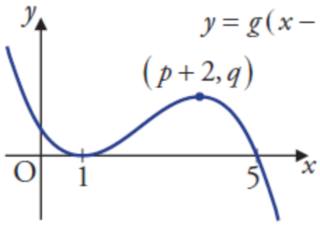
Translation parallel to the x-axis

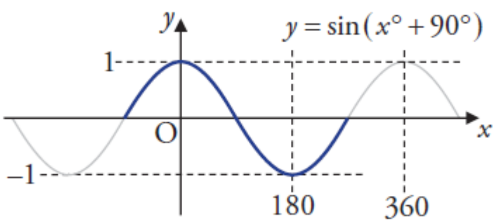
$f(x+a)$ moves the graph of $f(x)$ left or right. The graph is moved left if a is positive, and right if a is negative.

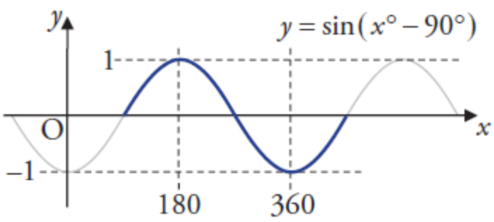
a is positive



a is negative







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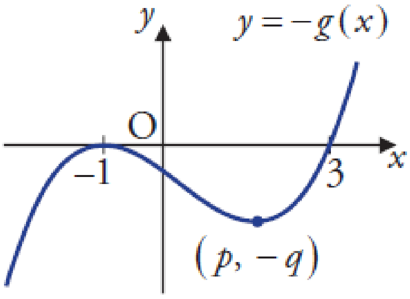
Reflection

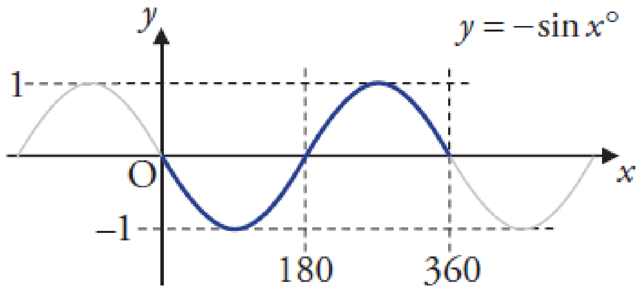
A **reflection** flips the graph about one of the axes.

When reflecting, the graph is flipped about one of the axes. It is important to apply this transformation *before* any translation.

Reflection in the x-axis

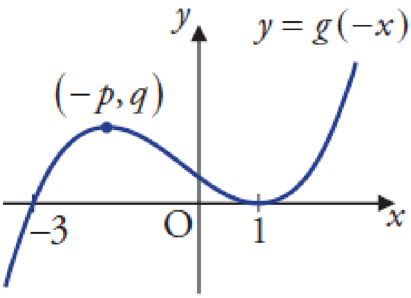
$-f(x)$ reflects the graph of $f(x)$ in the x-axis.

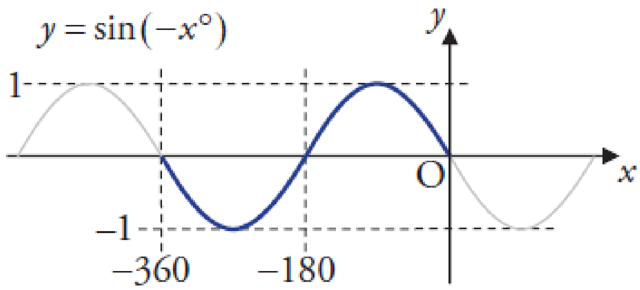




Reflection in the y-axis

$f(-x)$ reflects the graph of $f(x)$ in the y-axis.





From the graphs, $\sin(-x)°=-\sin x°$

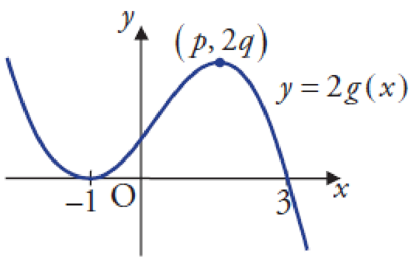
Scaling

A **scaling** stretches or compresses the graph along one of the axes.

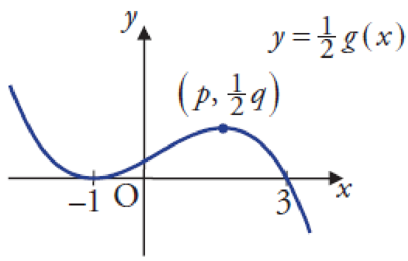
Scaling vertically

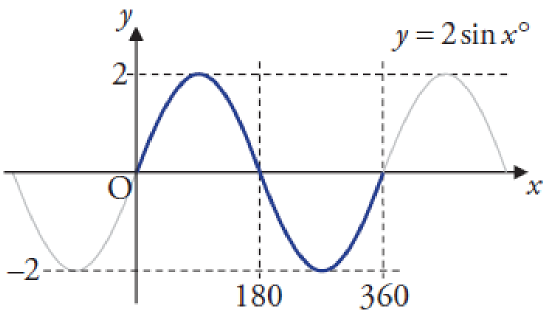
$kf(x)$ scales the graph of $f(x)$ in the vertical direction. The y-coordinate of each point on the graph is multiplied by k, roots are unaffected. These examples consider positive k.

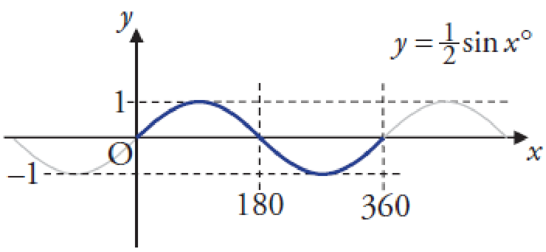
$k > 1$ stretches



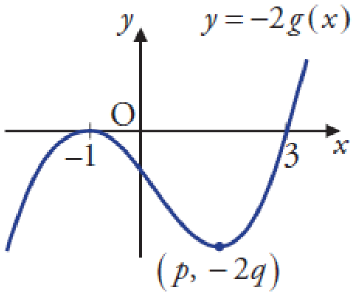
$0 < k < 1$ compresses







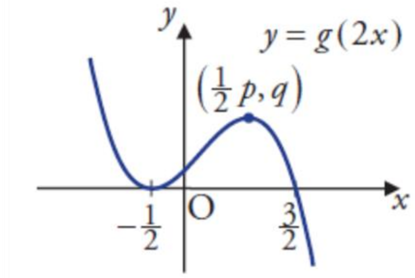
Negative k causes the same scaling, but the graph must then be reflected in the x-axis:



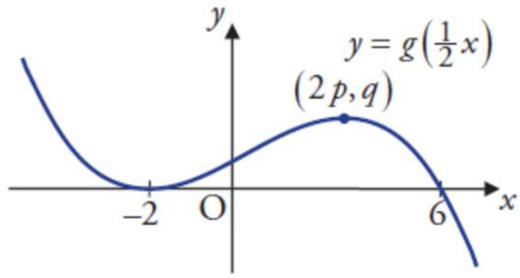
Scaling horizontally

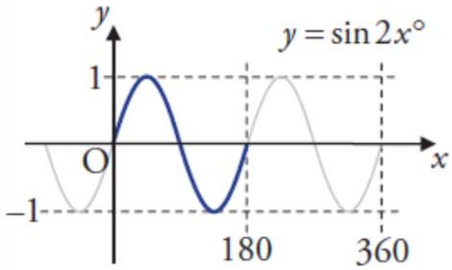
$f(kx)$ scales the graph of $f(x)$ in the horizontal direction. The coordinates of the y-axis intercept stay the same. The examples below consider positive k.

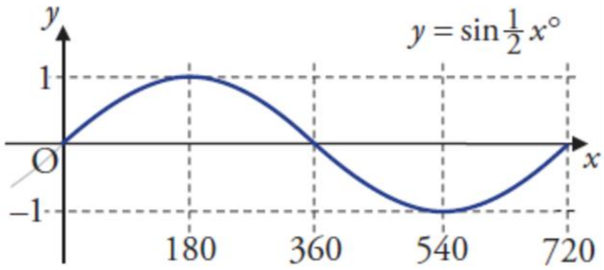
$k>1$



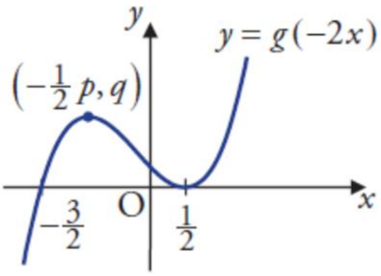
$0<k<1$





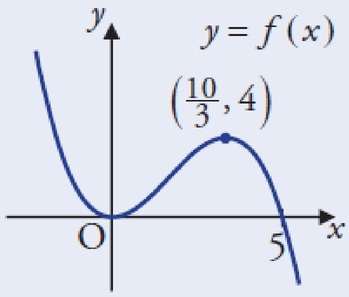


Negative k causes the same scaling, but the graph must then be reflected in the y-axis:



EXAMPLES

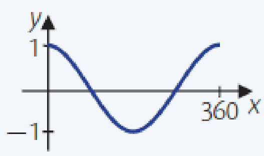
1. The graph of $y = f(x)$ is shown below.



Sketch the graph of $y = -f(x) - 2$.

2. Sketch the graph of $y = 5\cos(2x°)$ where $0 \leq x \leq 360$.

Remember
The graph of $y = \cos x$:



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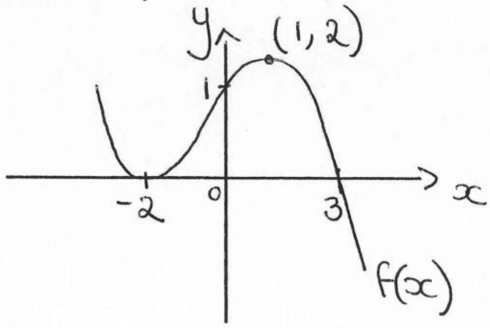
Examples

Shown is a graph of $y = f(x)$. On separate diagrams sketch

① $y = -f(x)$

② $y = f(-x) + 2$

③ $y = 1 - f(x)$



Completing the Square

The process of writing $y = ax^2 + bx + c$ in the form $y = a(x+p)^2 + q$ is called **completing the square**.

Once in "completed square" form we can determine the turning point of any parabola, including those with no real roots.

The axis of symmetry is $x = -p$ and the turning point is $(-p, q)$.

Step 1
Make sure the equation is in the form $y = ax^2 + bx + c$.

$y = 3x^2 + 12x - 3.$

Step 2
Take out the x^2-coefficient as a factor of the x^2 and x terms.

$y = 3(x^2 + 4x) - 3.$

Step 3
Replace the $x^2 + kx$ expression and compensate for the extra constant.

$$\begin{aligned} y &= 3\left((x+2)^2 - 4\right) - 3 \\ &= 3(x+2)^2 - 12 - 3. \end{aligned}$$

Step 4
Collect together the constant terms.

$y = 3(x+2)^2 - 15.$

Now that we have completed the square, we can see that the parabola with equation $y = 3x^2 + 12x - 3$ has turning point $(-2, -15)$.

EXAMPLES

1. Write $y = x^2 + 6x - 5$ in the form $y = (x+p)^2 + q$.

Note
You can always check your answer by expanding the brackets.

2. Write $x^2 + 3x - 4$ in the form $(x+p)^2 + q$.

3 Write $4x^2-12x+7$ in the form $a(x+b)^2+c$

4 Write $2x^2-10x+9$ in the form $a(x+b)^2+c$

5 Write $-4x^2-20x-13$ in the form $a(x+b)^2+c$

Sketching Parabolas

A parabola has equation $y = 4x^2 - 12x + 7$.

(a) Express the equation in the form $y = (x + a)^2 + b$.

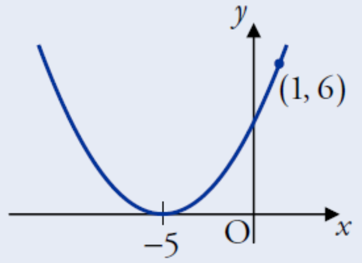
(b) State the turning point of the parabola and its nature.

2 Sketch the curve with equation $y = 2x^2 - 8x + 13$.

3 Sketch the curve with equation $y = -4x^2 - 10x + 6$

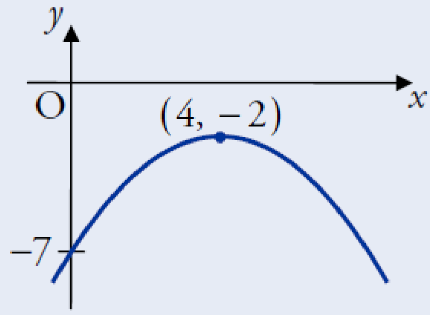
4

Find the equation of the parabola shown below.



5

Find the equation of the parabola shown below.



6 Show that $2x^2 - 6x + 33$ is positive for all values of x

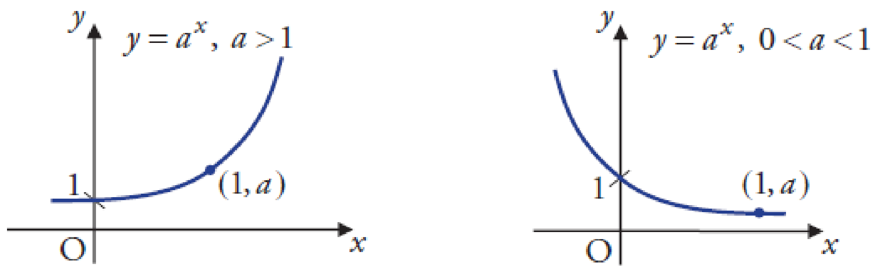
Exponential Functions EF

A function of the form $f(x) = a^x$ where $a, x \in \mathbb{R}$ and $a > 0$ is known as an **exponential** function to the base a.

We refer to x as the **power**, index or exponent.

Notice that when $x = 0$, $f(x) = a^0 = 1$. Also when $x = 1$, $f(x) = a^1 = a$.

Hence the graph of an exponential always passes through $(0, 1)$ and $(1, a)$:



EXAMPLE

Sketch the curve with equation $y = 6^x$.

2 a) Sketch the curve $y = 3^x$
b) Sketch the curve $y = 3^{x+1}$

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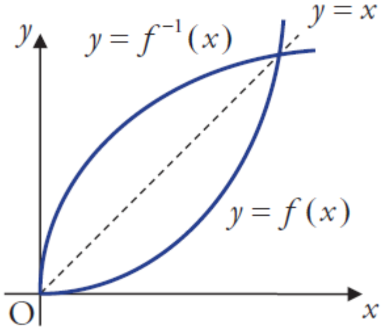
3 a) Sketch the curve $y = 3^x$
b) Sketch the curve $y = -3^x+2$

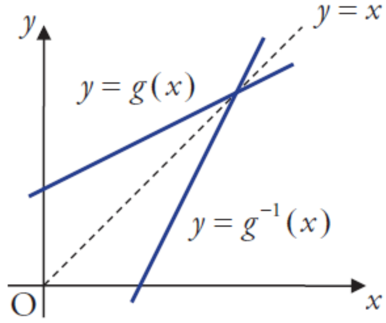
Logarithmic Graphs

Graphs of Inverses

If we have the graph of a function, then we can find the graph of its inverse by reflecting in the line $y = x$.

For example, the diagrams below show the graphs of two functions and their inverses.

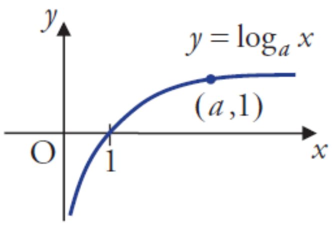




Logarithmic Functions

A **logarithmic** function is one in the form $f(x) = \log_a x$ where $a,\ x > 0$.

Logarithmic functions are inverses of exponentials, so to find the graph of $y = \log_a x$, we can reflect the graph of $y = a^x$ in the line $y = x$.



The graph of a logarithmic function always passes through $(1, 0)$ and $(a, 1)$.

EXAMPLE

Sketch the curve with equation $y = \log_6 x$.

Sketching Trigonometric Graphs

1 Sketch $y = 3\sin(x-30) + 2$

2 Sketch $y = 5\cos(x+\pi/3) - 4$

3 Sketch $y = 3\sin(3x - 60) + 2$

4 Sketch $y = 1 - 4\sin(3x + 30)$

Sets

In order to study functions and graphs, we use set theory. This requires some standard symbols and terms, which you should become familiar with.

A **set** is a collection of objects (usually numbers).

For example, $S = \{5,6,7,8\}$ is a set (we just list the object inside curly brackets).

We refer to the objects in a set as its **elements** (or members), e.g. 7 is an element of *S*. We can write this symbolically as $7 \in S$. It is also clear that 4 is *not* an element of *S*; we can write $4 \notin S$.

Given two sets *A* and *B*, we say *A* is a **subset** of *B* if all elements of *A* are also elements of *B*. For example, $\{6,7,8\}$ is a subset of *S*.

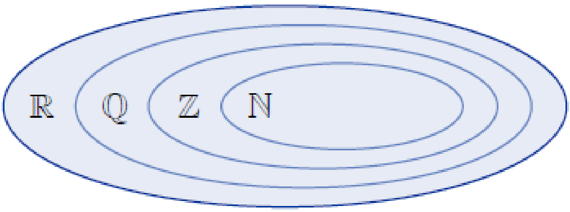
The **empty set** is the set with no elements. It is denoted by $\{\ \}$ or $\varnothing$.

Standard Sets

There are common sets of numbers which have their own symbols. Note that numbers can belong to more than one set.

$\mathbb{N}$	natural numbers	counting numbers, i.e. $\mathbb{N} = \{1, 2, 3, 4, 5, \ldots\}$.
$\mathbb{Z}$	integers	positive and negative whole numbers, i.e. $\mathbb{Z} = \{\ldots, -2, -1, 0, 1, 2, \ldots\}$.
$\mathbb{Q}$	rational numbers	can be written as a fraction of integers, e.g. $-4, \tfrac{1}{3}, 0{\cdot}25, -\tfrac{1}{3}$.
$\mathbb{R}$	real numbers	all points on the number line, e.g. $-6, -\tfrac{1}{2}, \sqrt{2}, \tfrac{1}{12}, 0{\cdot}125$.

Notice that $\mathbb{N}$ is a subset of $\mathbb{Z}$, which is a subset of $\mathbb{Q}$, which is a subset of $\mathbb{R}$. These relationships between the standard sets are illustrated in the "Venn diagram" below.



EXAMPLE

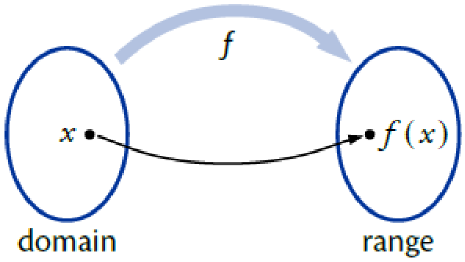
List all the numbers in the set $P = \{x \in \mathbb{N} : 1 < x < 5\}$.

Functions

EF

A **function** relates a set of inputs to a set of outputs, with each input related to exactly one output.

The set of inputs is called the **domain** and the resulting set of outputs is called the **range**.



A function is usually denoted by a lower case letter (e.g. f or g) and is defined using a formula of the form $f(x) = \ldots$. This specifies what the output of the function is when x is the input.

For example, if $f(x) = x^2 + 1$ then f squares the input and adds 1.

Restrictions on the Domain

The domain is the set of all possible inputs to a function, so it must be possible to evaluate the function for any element of the domain.

We are free to choose the domain, provided that the function is defined for all elements in it. If no domain is specified then we assume that it is as large as possible.

Division by Zero

It is impossible to divide by zero. So in functions involving fractions, the domain must exclude numbers which would give a denominator (bottom line) of zero.

For example, the function defined by:

$$f(x) = \frac{3}{x-5}$$

cannot have 5 in its domain, since this would make the denominator equal to zero.

The domain of f may be expressed formally as $\{x \in \mathbb{R} : x \neq 5\}$. This is read as "all x belonging to the real set such that x does not equal five".

Even Roots

Using real numbers, we cannot evaluate an even root (i.e. square root, fourth root etc.) of a negative number. So the domain of any function involving even roots must exclude numbers which would give a negative number under the root.

For example, the function defined by:

$$f(x) = \sqrt{7x-2}$$

must have $7x - 2 \geq 0$. Solving for x gives $x \geq \frac{2}{7}$, so the domain of f can be expressed formally as $\left\{x \in \mathbb{R} : x \geq \frac{2}{7}\right\}$.

EXAMPLE

1. A function g is defined by $g(x) = x - \dfrac{6}{x+4}$.

   Define a suitable domain for g.

1b A function *f* is defined by *f(x)* = $\sqrt{5x+1}$. Define a suitable domain for *f*

1c A function h is defined by h(x) = $\sqrt{2x-1} + \frac{7}{x-1}$

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Identifying the Range

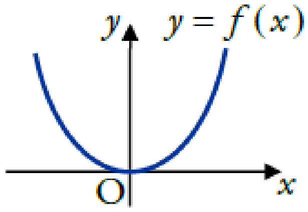
Recall that the range is the set of possible outputs. Some functions cannot produce certain values so these are not in the range.

For example:

$$f(x) = x^2$$

does not produce negative values, since any number squared is either positive or zero.

Looking at the graph of a function makes identifying its range more straightforward.



If we consider the graph of $y = f(x)$ (shown to the left) it is clear that there are no negative y-values.

The range can be stated as $f(x) \geq 0$.

Note that the range also depends on the choice of domain. For example, if the domain of $f(x) = x^2$ is chosen to be $\{x \in \mathbb{R} : x \geq 3\}$ then the range can be stated as $f(x) \geq 9$.

EXAMPLE

2. A function f is defined by $f(x) = \sin x^\circ$ for $x \in \mathbb{R}$. Identify its range.

2b A function g is defined by $g(x) = 2(x-1)^2 - 3$. Identify the range.

Composite Functions

Two functions can be "composed" to form a new **composite function.**

For example, if we have a squaring function and a halving function, we can compose them to form a new function. We take the output from one and use it as the input for the other.

$$x \longrightarrow \boxed{\text{square}} \longrightarrow x^2 \longrightarrow \boxed{\text{halve}} \longrightarrow \frac{x^2}{2}$$

The order is important, as we get a different result in this case:

$$x \longrightarrow \boxed{\text{halve}} \longrightarrow \frac{x}{2} \longrightarrow \boxed{\text{square}} \longrightarrow \frac{x^2}{4}$$

Using function notation we have, say, $f(x) = x^2$ and $g(x) = \frac{x}{2}$.

The diagrams above show the composite functions:

$$g(f(x)) = g(x^2) = \frac{x^2}{2} \qquad\qquad f(g(x)) = f\left(\frac{x}{2}\right) = \left(\frac{x}{2}\right)^2 = \frac{x^2}{4}.$$

EXAMPLES

1. Functions f and g are defined by $f(x) = 2x$ and $g(x) = x - 3$. Find:

   (a) $f(2)$ (b) $f(g(x))$ (c) $g(f(x))$

2. Functions f and g are defined on suitable domains by $f(x)=x^3+1$ and $g(x)=\frac{1}{x}$.

   Find formulae for $h(x)=f(g(x))$ and $k(x)=g(f(x))$.

Exam Question

A function f is defined on the set of real numbers by $f(x)=\dfrac{x}{1-x}$, $x \neq 1$.

Find, in its simplest form, an expression for $f(f(x))$. **3**

Inverse Functions

EF

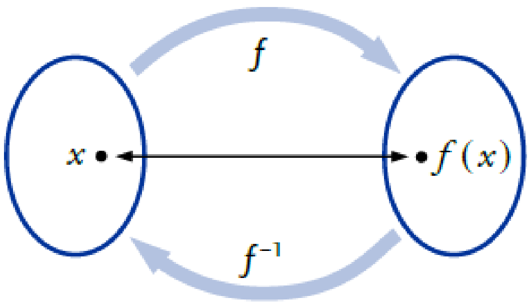
The idea of an **inverse function** is to reverse the effect of the original function. It is the "opposite" function.

You should already be familiar with this idea – for example, doubling a number can be reversed by halving the result. That is, multiplying by two and dividing by two are inverse functions.

The inverse of the function f is usually denoted f^{-1} (read as "f inverse").

The functions f and g are said to be **inverses** if $f(g(x)) = g(f(x)) = x$.

This means that when a number is worked through a function f then its inverse f^{-1}, the result is the same as the input.



For example, $f(x) = 4x - 1$ and $g(x) = \dfrac{x+1}{4}$ are inverse functions since:

$$\begin{aligned} f(g(x)) &= f\left(\frac{x+1}{4}\right) \\ &= 4\left(\frac{x+1}{4}\right) - 1 \\ &= x + 1 - 1 \\ &= x \end{aligned}$$

$$\begin{aligned} g(f(x)) &= g(4x-1) \\ &= \frac{(4x-1)+1}{4} \\ &= \frac{4x}{4} \\ &= x. \end{aligned}$$

Formulae for Inverses

The example below shows how to find a formula for the inverse of a function.

EXAMPLES

1. A function f is defined, for all real numbers, by $f(x) = x^3 + 1$.
   Find a formula for its inverse f^{-1}.

Step 1
Replace $f(x)$ with y in the formula.

Step 2
Change the subject of this formula to x.

Step 3
Interchange x and y.

Step 4
Replace y with $f^{-1}(x)$ to obtain the formula for the inverse.

2. A function g is defined, for all real numbers, by $g(x) = \dfrac{x-3}{2}$.
   Find a formula for its inverse g^{-1}.

Step 1
Replace $f(x)$ with y in the formula.

Step 2
Change the subject of this formula to x.

Step 3
Interchange x and y.

Step 4
Replace y with $f^{-1}(x)$ to obtain the formula for the inverse.

3 Given that f(2) = 7 find $f^{-1}(7)$

4. $f(x) = 2x - 1$, $g(x) = 3 - 2x$ and $h(x) = \frac{1}{4}(5 - x)$.

(*a*) Find a formula for $k(x)$ where $k(x) = f\big(g(x)\big)$. **2**

(*b*) Find a formula for $h\big(k(x)\big)$. **2**

(*c*) What is the connection between the functions h and k? **1**

Extension to Logarithmic and Exponential Functions

1 State the range of the following functions:

a) $f(x) = 4^x - 7$ b) $f(x) = 3^{x+1} + 3$

2 State the domain of the following functions:

a) $g(x) = \log_3(x - 4)$ b) $h(x) = \log_3(2x + 5) - 3$

3 Given that $f(x) = 4^x$ and $g(x) = x + 2$ express $f((g(x))$ in the form $A(4^x)$ where A is a constant.

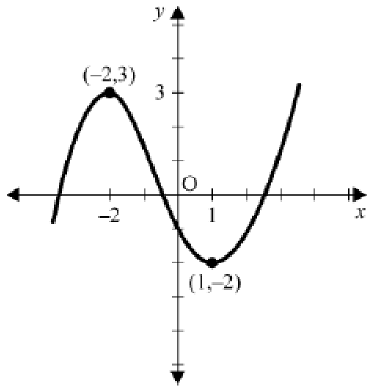
Practice unit assessment questions

1 Sketch the graph of $a\cos(x-\frac{\pi}{3})$ for $0 \le x \le 2\pi$ and $a > 0$.

Show clearly the intercepts on the x-axis and the coordinates of the turning points.

[4]

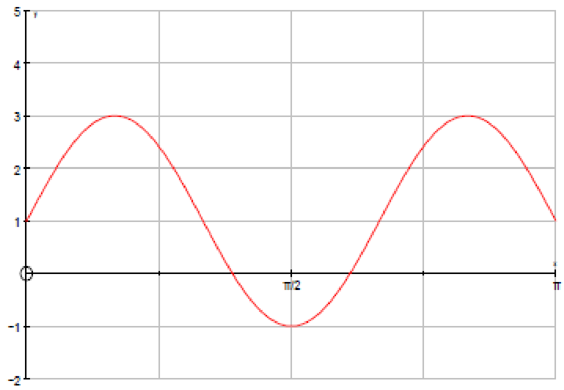
2 The diagram shows the graph of $y = f(x)$ with a maximum turning point at (–2, 3) and a minimum turning point at (1, –2).



Sketch the graph of $y = f(x+2)-1$

[3]

3 The diagram below shows the graph of $y = a\sin(bx)+c$.

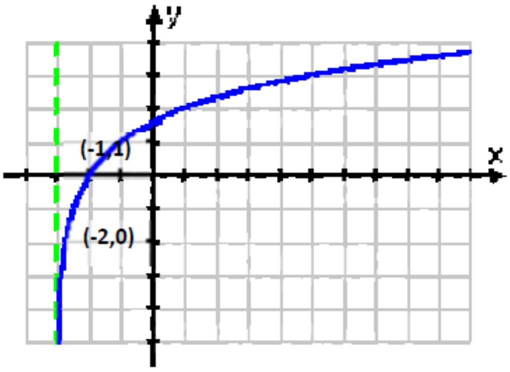


Write down the values of a, b and c.

[3]

Additional examples are available in the Dynamic Maths Study Notes available at http://madrasmaths.com/courses/higher/revison_materials_higher.html (password: madrasmaths) and at hsn.uk.net

4 The diagram shows the graph of $y = \log_b(x-a)$.



Determine the values of *a* and *b*. [2]

5 The functions f and g, defined on suitable domains contained within the set of real numbers, $f(x) = 5x - 2$, $g(x) = \sqrt{x-1}$. A third function *h*(*x*) is defined as $h(x) = g(f(x))$.

(a) Find an expression for $h(x)$.

[2]

(b) Explain why x = 0 is not in the domain of $h(x)$.

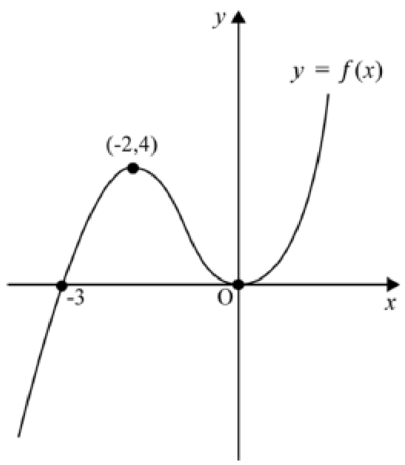
[#2.2]

6 A function is given by $f(x) = 4x + 6$. Find the inverse function $f^{-1}(x)$.

[3]

Practice test 2

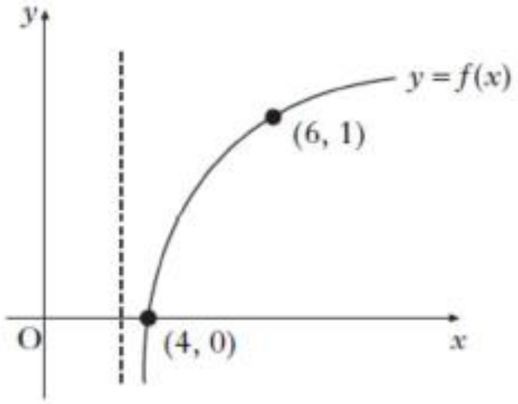
1 The diagram shows the graph of $y = f(x)$ with a maximum turning point at (–2, 4) and a minimum turning point at (0, 0).



Sketch the graph of $y = 1 - f(x-3)$.

[4]

2 The diagram shows the graph of $y=\log_b(x-a)$.



Determine the values of a and b.

[2]

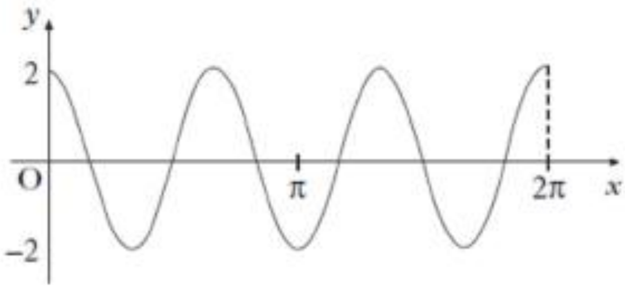
3 Sketch the graph of $y=3\cos(x+\frac{\pi}{4})$ for $0\le x\le 2\pi$.

Show clearly the intercepts on the x-axis and the coordinates of the turning points.

[4]

4 The diagram shows the graph of $y = a\cos(bx)$ for $0\le x\le 2\pi$.

State the values of a and b.



[2]

5 The functions f and g are defined on suitable domains contained within the set of real numbers,

$f(x)=\dfrac{1}{x^2-16}$ and $g(x)=x-2$.

A third function, h, is defined as $h(x)=f(g(x))$.

(a) Find an expression for $h(x)$.

[2]

(b) Find a suitable domain for $h(x)$.

[#2.1 + 2]

6 A function is given by $f(x)=2-\sqrt[3]{x}$. Find the inverse function $f^{-1}(x)$.

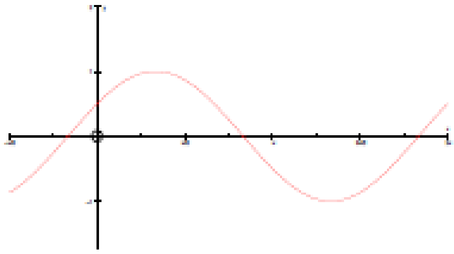
[3]

Additional examples are available in the Dynamic Maths Study Notes available at http://madrasmaths.com/courses/higher/revison_materials_higher.html (password: madrasmaths) and at hsn.uk.net

Answers to Practice Tests

Practice test 1

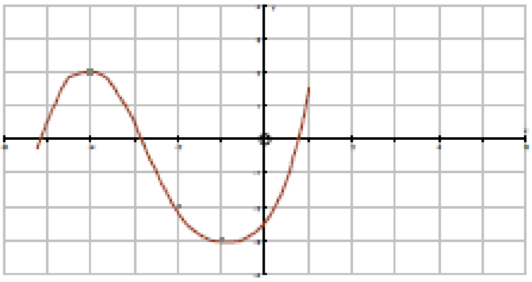
1



x-intercepts: $\left(\frac{5\pi}{6},0\right)$ and $\left(\frac{11\pi}{6},0\right)$

Max = a $\left(\frac{\pi}{3},a\right)$ Min = -a $\left(\frac{4\pi}{3},a\right)$

2



(-4,2), (-1,-3) and (-2,-2) clearly annotated

3 $a = 2, b = 3, c = 1$

4 $a = -3, b = 2$

5 (a) $h(x) = \sqrt{5x-3}$

(b)
$$5x-3 \geq 0$$
$$5x \geq 3$$
$$x \geq \frac{3}{5}$$

Because on the set of real numbers, the square root of a negative number cannot be found.

6 $f^{-1}(x) = \frac{1}{4}(x-6)$

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Practice test 2

1
- Horizontal translation (3 units to the right)
- Reflection in the *x*-axis
- Vertical translation (1 unit up)
- Each image annotated clearly: (-3, 0) to (0, 1)
  (-2, 4) to (1, -1)
  (0, 0) to (3, 1)

2 *a* = 3, *b* = 3

3
- Amplitude correct: Max value = 3, Min value = -3
- Correct turning points: Min. T.P. $\left(\frac{3\pi}{4},\ -3\right)$ Max T.P. $\left(\frac{7\pi}{4},\ 3\right)$
- Correct x-intercepts $\left(\frac{\pi}{4},\ 0\right)\left(\frac{5\pi}{4},\ 0\right)$
- Correct shape, i.e. cosine curve

  Note: y-intercept = $\frac{3}{\sqrt{2}}=\frac{3\sqrt{2}}{2}=2\cdot121\ldots$

4 *a* = 2, *b* = 3

5 (a) $h(x)=\frac{1}{(x-2)^2-16}$

(b)
$$(x-2)^2-16=0$$
$$(x-2)^2=16$$
$$x-2=\pm\ 4$$
$$x=-2,\ 6$$
domain = $\{x\in R : x\neq -2, x\neq 6\}$

6 $f^{-1}(x)=(2-x)^3$

Homework 3 – Expressions and Functions

Non-Calculator

Functions f and g are defined on $\mathbb{R}$, the set of real numbers.
The inverse functions f^{-1} and g^{-1} both exist.

(a) Given $f(x) = 3x + 5$, find $f^{-1}(x)$. **3**

(b) If $g(2) = 7$, write down the value of $g^{-1}(7)$. **1**

SQA Higher Maths 2016 Non Calc Q6

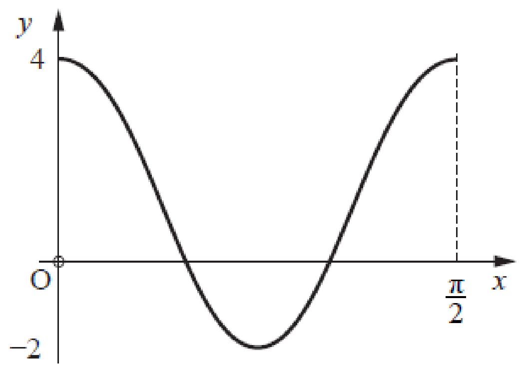
The functions f and g are defined on $\mathbb{R}$, the set of real numbers by
$f(x) = 2x^2 - 4x + 5$ and $g(x) = 3 - x$.

(a) Given $h(x) = f(g(x))$, show that $h(x) = 2x^2 - 8x + 11$. **2**

(b) Express $h(x)$ in the form $p(x+q)^2 + r$. **3**

SQA Higher Maths 2016 Non Calc Q12

The diagram shows part of the graph of the function $y = p\cos qx + r$.



Write down the values of p, q and r. **3**

SQA Higher Maths 2015 Non Calc Q4

A function g is defined on $\mathbb{R}$, the set of real numbers, by $g(x) = 6 - 2x$.

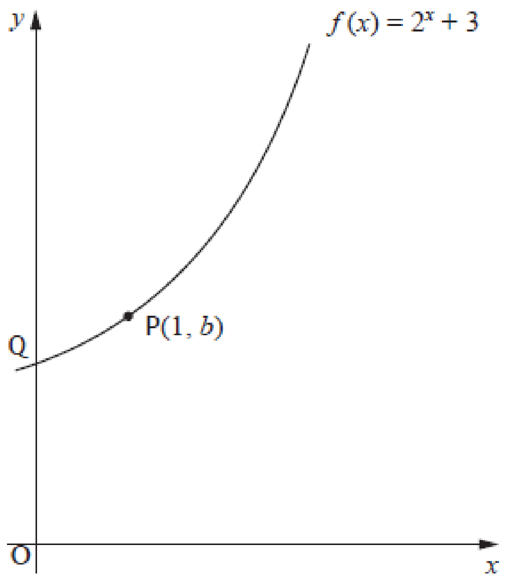
(a) Determine an expression for $g^{-1}(x)$. **2**

(b) Write down an expression for $g(g^{-1}(x))$. **1**

SQA Higher Maths 2015 Non Calc Q5

The function $f(x) = 2^x + 3$ is defined on $\mathbb{R}$, the set of real numbers.

The graph with equation $y = f(x)$ passes through the point P$(1, b)$ and cuts the y-axis at Q as shown in the diagram.



(a) What is the value of b? **1**

(b) (i) Copy the above diagram.

On the same diagram, sketch the graph with equation $y = f^{-1}(x)$. **1**

(ii) Write down the coordinates of the images of P and Q. **3**

(c) R $(3,11)$ also lies on the graph with equation $y = f(x)$.

Find the coordinates of the image of R on the graph with equation $y = 4 - f(x + 1)$. **2**

SQA Higher Maths 2015 Non Calc Q13

Calculator Section

Functions f and g are defined on suitable domains by

$$f(x) = 10 + x \text{ and } g(x) = (1+x)(3-x)+2.$$

(a) Find an expression for $f(g(x))$. **2**

(b) Express $f(g(x))$ in the form $p(x+q)^2+r$. **3**

(c) Another function h is given by $h(x) = \dfrac{1}{f(g(x))}$.

What values of x cannot be in the domain of h? **2**

SQA Higher Maths 2015 Calc Q2

Additional examples are available in the Dynamic Maths Study Notes available at http://madrasmaths.com/courses/higher/revison_materials_higher.html (password: madrasmaths) and at hsn.uk.net